

A NOTE ON ANALYTIC FUNCTIONS DEFINED BY INTEGRAL OPERATOR

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Abstract

Let A denote the class of functions of the form $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ which are analytic in the open unit disc $D = \{z : |z| < 1\}$. The subclass of A consisting of normalized univalent functions is denoted by S . The subclasses of S , which we denote by S^* , C and K consists of starlike, convex and close to convex functions, respectively. It is well known that $C \subset S^* \subset K \subset S$. For $f \in A$, Jung et al. have introduced the integral operator $J_{\alpha} f$. Let T denote subclass of S consisting of

functions f of the form $f(z) = z - \sum_{n=2}^{\infty} a_n z^n$ where a_n a non negative real number. For

$f \in T$, we define $J_{\alpha} f(z) = z - \sum_{n=2}^{\infty} \frac{\alpha + 1}{\alpha + n} a_n z^n$ where $\alpha > -1$. Using the above operator $J_{\alpha} f$, we now introduce a new class of functions as a result of the combination of ideas given by Jung et al. and Rusli and Darus. We define the following class $M_{\alpha}(\sigma, k)$.

Let $f \in T$. Then $f \in M_{\alpha}(\sigma, k)$ if and only if

$$\left| \frac{\frac{zJ'(z)}{J(z)} - k}{\frac{zJ'(z)}{J(z)} - (2\sigma - k)} \right| < 1$$

where $J(z) = J_\alpha f(z)$, $\alpha > -1$, $\sigma < k \leq 1$ and $z \in D$. This paper gives some interesting properties, namely coefficient and growth results pertaining to functions f and belongs to the class $M_\alpha(\sigma, k)$.

Keywords: Analytic, coefficient estimates, growth